Københavns Universitets Økonomiske Institut
Asset Pricing Theory **December 16, 2009**

Three hour written, closed book examination
Two pages with three questions. The weights are only indicative.

Question 1 (35 %).

(1) Describe the one-period model under uncertainty. Explain the concepts of state space, securities, prices, dividend matrix and portfolio.

(2) Introduce the notions of arbitrage and state price vector. Show that the existence of a state price vector implies the absence of arbitrage.

(3) Introduce the notion of risk neutral probabilities and assume the existence of a zero coupon bond. Show that asset prices are the discounted expected dividends with respect to the risk neutral distribution.

Question 2 (35 %).

(1) Describe the Black-Scholes-Merton model and list the main assumptions.

(2) Derive Black-Scholes-Merton's differential equation.

(3) Derive the Feyman-Kac solution to the pricing of a derivative in Black-Scholes-Merton model.

Question 3 (30 %). Consider a bond market with term structure $P(t,T)$.

(1) Define the zero coupon yield $y(t,T)$ and show that it is given by the formula

$$y(t,T) = -\frac{\log P(t,T)}{T-t}$$

for $0 \leq t \leq T$.

We seek to write a contract at time t which allows us to make an investment of one unit at time T_1 and to have a deterministic rate of return, determined at time t, over the interval $[T_1, T_2]$ where $0 \leq t < T_1 < T_2 \leq T$.

(2) Show that we may achieve this aim by selling one T_1-bond at time t and use the amount to buy T_2-bonds.

The equivalent continuously compounded forward rate contracted at time t is denoted by $f(t, T_1, T_2)$.

(3) Determine a formula for $f(t, T_1, T_2)$ in terms of zero coupon bond prices and calculate the instantaneous forward rate $f(t, T)$.